Example 10: Consider the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$. Is A invertible? Explain. Calculate $\det(A)$.

$A \underset{R_2 := R_2 - 4R_1}{\overset{R_3 := R_3 - 7R_1}{\sim}} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{bmatrix} \overset{R_3 := R_3 - 2R_2}{\sim} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix} = C$

Since rank$(A) = 2$, A is not invertable by FTIM.

$\det(A) = \det(C) = 0.$

If A and C are row equivalent,

either ① $\det(A) \neq 0$ and $\det(C) \neq 0$

② $\det(A) = 0$, and $\det(C) = 0$

Proposition 1: If a $n \times n$ matrix A is not invertible, then $\det(A) = 0$.

Theorem 4.6: A $n \times n$ matrix A is invertible if and only if $\det(A) \neq 0$.

Proof: case 1] If A is not invertable, then $\det(A) = 0$ by proposition 1.

case 2] If A is invertable, then

$AA^{-1} = I_n$

$1 = \det(I_n) = \det(AA^{-1}) = \det(A)\det(A^{-1})$

$\det(A) \neq 0$